Passive control theory I

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II EURON/GEOPLEX Summer School on

Modeling and Control of Complex Dynamical Systems

Bertinoro, Italy, July 18-22 2005

UNIVERSITAT POLITÈCNICA DE CATALUNYA

Contents of this lecture

Change of paradigm in control: from signal based to energy based

Fits well with the PHDS modeling approach

Energy balance control

Control as interconnection

Casimir functions and the dissipation obstacle

References

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Energy based control

Control problems have been approached traditionally adopting a signal-processing viewpoint.

Very useful for linear time-invariant systems, where signals can be discriminated via filtering.

For nonlinear systems, frequency mixing makes things harder:

very involved computations

very complex and energy comsuming controls are needed to suppress the large set of undesirable signals

Most of the problem stems from not using
any information about the physical structure of the system.

We have learnt from the previous lectures that
energy plays an essential role in the description of physical systems.

$\Downarrow$

energy based control

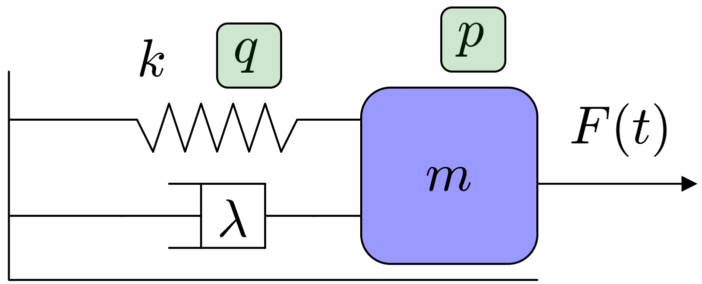
the controller should shape the energy of the system,
and even change how energy flows inside the system.

Passivity and energy balance control

Consider a system with states $x \in \mathbb{R}^n$, inputs $u \in \mathbb{R}^m$ and outputs $y \in \mathbb{R}^m$

The map $u \mapsto y$ is *passive* if there exists a state function $H(x)$, bounded from below, and a nonnegative function $d(t) \geq 0$ such that

$$\underbrace{\int_0^t u^T(s)y(s)\,\mathrm{d}s}_{\text{energy supplied to the system}} = \underbrace{H(x(t)) - H(x(0))}_{\text{stored energy}} + \underbrace{d(t)}_{\text{dissipated}}$$



$$u = F,\ y = v = p/m$$

$$\begin{aligned} \dot{q} &= \frac{p}{m} \\ \dot{p} &= F(t) - kq - \lambda \frac{p}{m} \end{aligned}$$

$$\int_0^t u(s)y(s)\ \mathrm{d}s = \int_0^t F(s)v(s)\ \mathrm{d}s = \int_0^t \left(\dot{p}(s) + kq(s) + \frac{\lambda}{m} p(s) \right) \frac{p(s)}{m}\ \mathrm{d}s$$

$$= \left(\frac{1}{2m} p^2(s) + \frac{1}{2} kq^2(s) \right) \Bigg|_{s=0}^{s=t} + \frac{\lambda}{m^2} \int_0^t p^2(s)\ \mathrm{d}s$$

$$= H(x(t)) - H(x(0)) + \boxed{\frac{\lambda}{m^2} \int_0^t p^2(s)\ \mathrm{d}s}$$

$$\boxed{H(x) = \frac{1}{2m} p^2 + \frac{1}{2} kq^2}$$

$$d(s) \geq 0$$

Consider an explicit PHDS

$$\begin{aligned} \dot{x} &= (J(x) - R(x))\partial_x H(x) + g(x)u \\ y &= g^T(x)\partial_x H(x) \end{aligned} \qquad \begin{aligned} J^T &= -J \\ R^T &= R \geq 0 \end{aligned}$$

$$\begin{aligned} \int_0^t u^T y \,\mathrm{d}\tau &= \int_0^t u^T g^T \partial_x H \,\mathrm{d}\tau = \int_0^t (gu)^T \partial_x H \,\mathrm{d}\tau \\ &= \int_0^t \left(\dot{x}^T + (\partial_x H)^T J + (\partial_x H)^T R\right) \partial_x H \,\mathrm{d}\tau \\ &= \int_0^t \partial_\tau H \,\mathrm{d}\tau + \int_0^t (\partial_x H)^T R \partial_x H \,\mathrm{d}\tau \\ &= H(x(t)) - H(x(0)) + \underbrace{\int_0^t (\partial_x H)^T R \partial_x H \,\mathrm{d}\tau}_{\geq 0} \end{aligned}$$

does not depend on the PHDS being explicit!

PHDS are passive for $u \mapsto y$ and storage function H

If x^* is a global minimum of $H(x)$ and $d(t) > 0$, and we set $u = 0$, $H(x(t))$ will decrease in time and the system will reach x^* asymptotically.

The rate of convergence can be increased if we actually extract energy from the system with $u = -K_{\mathrm{di}}\ y$ with $K_{\mathrm{di}}^T = K_{\mathrm{di}} > 0$

If $d \geq 0$ only, then invariant sets have to be examined and LaSalle theorem has to be invoked.

However, the minimum of the energy of the system is not a very interesting point from an engineering perspective.

PHYSICS

As engineers, we are not really concerned in knowing what Nature does, but rather in forcing Nature to do what we want.

Key idea of passivity based control (PBC)

use feedback $u(t) = \beta(x(t)) + v(t)$

so that the closed-loop system is again a passive system,
with energy function H_{d}, with respect to $v \mapsto y$,
and such that H_{d} has the global minimum at the desired point.

If $$\boxed{-\int_0^t \beta^T(x(s))y(s)\ \mathrm{d}s} = H_{\mathrm{a}}(x(t))$$

Why should this be a state function?

then the closed loop system is passive (with input v) and has energy function

$$H_d(x) = H(x) + H_a(x)$$

prove that

If the preceding assumptions hold, then

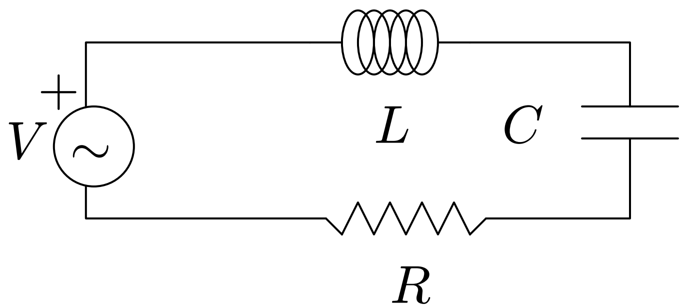
$$\underbrace{H_{\mathrm{d}}(x(t))}_{\text{closed-loop energy}} = \underbrace{H(x(t))}_{\text{stored energy}} - \underbrace{\int_0^t \beta^T(x(s))y(s)\,\mathrm{d}s}_{\text{supplied energy}}$$

This is called the Energy Balance Equation (EBE)

For $\dot{x} = f + gu$, $y = h$ systems, the EBE is equivalent to the PDE

$$-\beta^T(x)h(x) = \left(\frac{\partial H_{\mathrm{a}}}{\partial x}(x)\right)^T (f(x) + g(x)\beta(x))$$

As an example, consider the RLC circuit



$$x = \begin{pmatrix} q \\ \phi \end{pmatrix} \quad \text{state}$$

$$u = V \quad \text{control}$$

$$y = i = \frac{\phi}{L} \quad \text{output}$$

$$\dot{x} = \begin{pmatrix} x_2/L \\ -x_1/C - x_2R/L \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} V$$

The map $V \mapsto i$ is passive with energy function

$$H(x) = \frac{1}{2C}x_1^2 + \frac{1}{2L}x_2^2$$

and dissipation $d(t) = \int_0^t \frac{R}{L^2}\phi^2(s)\ \mathrm{d}s$

If we set $V = 0$, the natural equilibria is $(0, 0)$

For $V = V^*$, the *forced* equilibria are $(x_1^*, 0)$, with $x_1^* = CV^*$

The PDE for energy balance control to be solved is

$$\frac{x_2}{L}\frac{\partial H_{\mathrm{a}}}{\partial x_1} - \left(\frac{x_1}{C} + \frac{R}{L}x_2 - \beta(x)\right)\frac{\partial H_{\mathrm{a}}}{\partial x_2} = -\frac{x_2}{L}\beta(x)$$

Do we really have to solve this?

The natural energy already has a minimum at the desired forced equilibria $x_2^* = 0$

⇨ we only need to shape the energy with respect to x_1 ⇨ we can take H_a as a function of x_1 only

$$\beta(x_1) = -\frac{\partial H_{\mathrm{a}}}{\partial x_1}(x_1)$$

No equation at all: it defines the control!

The only remaining task is to choose H_a so that H_d has the minimum at $(x_1^*, 0)$.

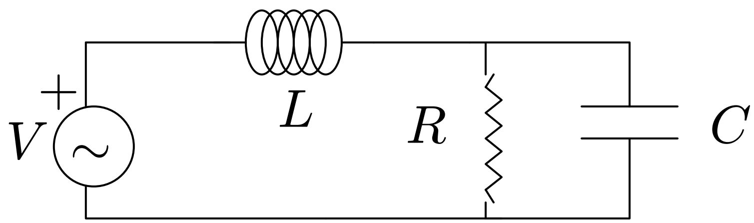
$$H_{\mathrm{a}}(x_1) = \frac{1}{2C_a}x_1^2 - \left(\frac{1}{C} + \frac{1}{C_a}\right)x_1^* x_1$$

C_a is a design parameter to be tuned for performance

It can be seen that the resulting H_d has a minimum at $(x_1^*, 0)$ iff $C_a > -C$

$$u = -\frac{\partial H_{\mathrm{a}}}{\partial x_1}(x_1) = -\frac{x_1}{C_a} + \left(\frac{1}{C} + \frac{1}{C_a}\right)x_1^*$$

Consider now this slightly different circuit



$$\begin{aligned}\dot{x}_1 &= -\frac{1}{RC}x_1 + \frac{1}{L}x_2,\\ \dot{x}_2 &= -\frac{1}{C}x_1 + V(t)\end{aligned}$$

Only the dissipation structure has changed, but the admissible equilibria are of the form

$$x_1^* = CV^*, \quad x_2^* = \frac{L}{R}V^*$$

The power delivered by the source, Vx_2/L, is nonzero at any equilibrium point except for the trivial one

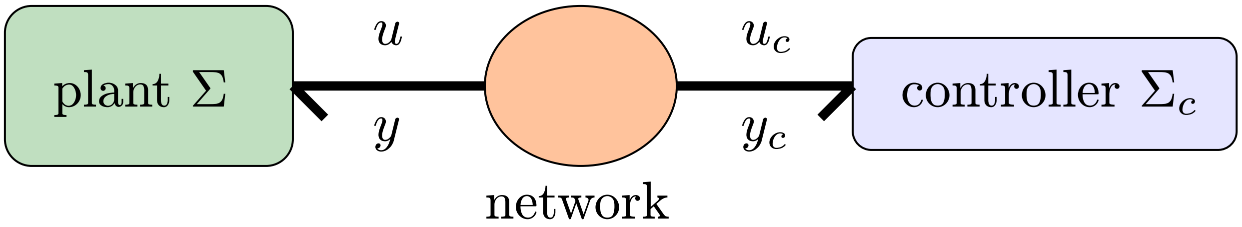
the source has to provide an infinite amount of energy to keep any nontrivial equilibrium point

this is the infamous dissipation obstacle, which will reappear later

Control as interconnection

We would like to have a physical interpretation of the preceding ideas

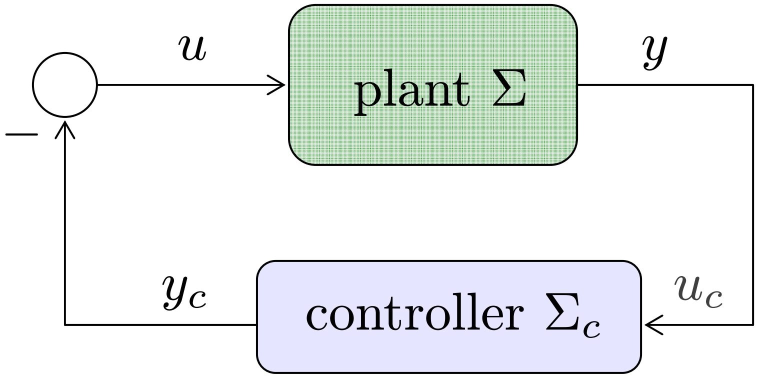
Plant and controller as two physical systems exchanging energy over a network



The interconnection is power continuous if

$$u_c^T(t)y_c(t) + u^T(t)y(t) = 0 \quad \forall t$$

As an example, consider the standard feedback interconnection

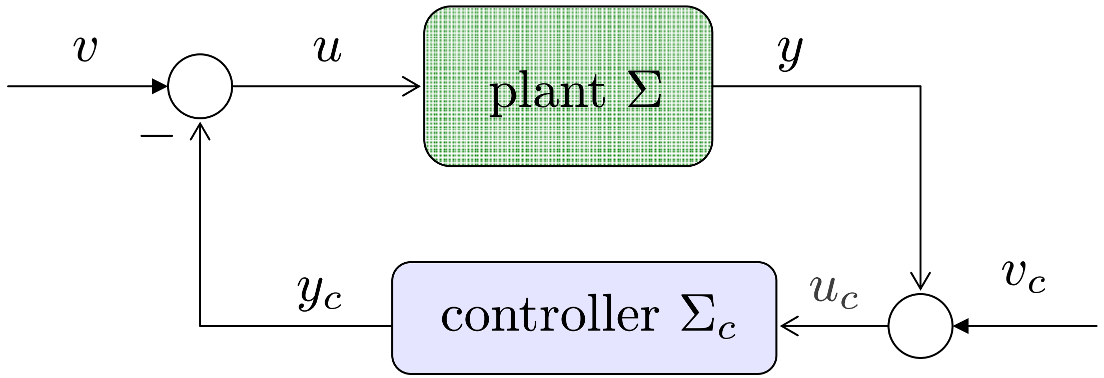


$$
\begin{aligned}
u_c &= y \\
u &= -y_c
\end{aligned}
$$

It is clearly power continuous

$$u_c y_c + u y = y y_c + (-y_c) y = 0$$

Assume we have one of such interconnections, and add extra inputs $u \to u + v$, $u_c \to u_c + v_c$



Let Σ and Σ_c have state variables x and ξ, and let the maps $u \mapsto y$ and $u_c \mapsto y_c$ be passive, with energy functions $H(x)$ and $H_c(\xi)$, respectively.

Then

the map $(v, v_c) \mapsto (y, y_c)$ is passive for the interconnected system, with energy function $H_d(x, \xi) = H(x) + H_c(\xi)$.

power continuous interconnection of passive systems yields passive systems

We have a passive system with state variables (x, ξ)
and energy function $H_d(x, \xi) = H(x) + H_c(\xi)$.

We would like to get an energy function H_d
in terms of x only, so that we can set the minimum at the desired point.

In order to do this, we must **restrict** the dynamics to a submanifold
of the (x, ξ) space parameterized by x, so that

$$\Omega_K = \{(x, \xi) \; ; \; \xi = F(x) + K\}$$

level constant

is dinamically **invariant** $$\left(\left(\frac{\partial F}{\partial x} \right)^T \dot{x} - \dot{\xi} \right)_{\xi = F(x)+K} = 0.$$

This can be formulated better in the PHDS framework

Casimir functions and the dissipation obstacle

Suppose both the plant and the controller are PHDS

$$\Sigma : \begin{cases} \dot{x} &= (J(x) - R(x))\frac{\partial H}{\partial x}(x) + g(x)u \\ y &= g^T(x)\frac{\partial H}{\partial x}(x) \end{cases}$$

$$\Sigma_c : \begin{cases} \dot{\xi} &= (J_c(\xi) - R_c(\xi))\frac{\partial H_c}{\partial \xi}(\xi) + g_c(\xi)u_c \\ y_c &= g_c^T(\xi)\frac{\partial H_c}{\partial \xi}(\xi) \end{cases}$$

For simplicity, take the standard feedback interconnection $u = -y_c$, $u_c = y$

$$\begin{pmatrix} \dot{x} \\ \dot{\xi} \end{pmatrix} = \begin{pmatrix} J(x) - R(x) & -g(x)g_c^T(\xi) \\ g_c(\xi)g^T(x) & J_c(\xi) - R_c(\xi) \end{pmatrix} \begin{pmatrix} \partial_x H_d \\ \partial_\xi H_d \end{pmatrix}$$

with $H_d(x, \xi) = H(x) + H_c(\xi)$

Let us look for invariant manifolds of the form

$$C_K(x,\xi) = F(x) - \xi + K$$

Condition $\dot{C}_k = 0$ yields

$$\begin{pmatrix} \left(\frac{\partial F}{\partial x}\right)^T & | & -\mathbb{I} \end{pmatrix} \begin{pmatrix} J - R & -gg_c^T \\ g_c g^T & J_c - R_c \end{pmatrix} \begin{pmatrix} \partial_x H_d \\ \partial_\xi H_d \end{pmatrix} = 0$$

since $H_d = H + H_a$ and we want total freedom in choosing H_a, we cannot count on the gradient of H_d, so

$$\begin{pmatrix} \left(\frac{\partial F}{\partial x}\right)^T & | & -\mathbb{I} \end{pmatrix} \begin{pmatrix} J - R & -gg_c^T \\ g_c g^T & J_c - R_c \end{pmatrix} = 0$$

This is a PDE for $F(x)$

Functions $C_K(x,\xi) = F(x) - \xi + K$ such that F satisfies the above PDE on $C_K = 0$ are called Casimir functions.

They are invariants associated to the structure of the system (J, R, g, J_c, R_c, g_c), independently of the Hamiltonian function

It can be shown that the PDE for F has solution iff, on $C_K = 0$,

1. $(\partial_x F)^T J \partial_x F = J_c$
2. $R \partial_x F = 0$
3. $R_c = 0$
4. $(\partial_x F)^T J = g_c g^T$

No dissipation in the variables on which the Casimir depends (it must be invariant!)

$R\partial_x F = 0$ is the dissipation obstacle again.

If a Casimir can be found, the closed loop dynamics on C_K is given by

$$\dot{x} = (J(x) - R(x))\frac{\partial H_d}{\partial x} \qquad \text{with } H_d(x) = H(x) + H_c(F(x) + K)$$

while the ξ evolve as their $x-$parametrization dictates.

No dissipation obstacle appears in mechanical systems,
since dissipation only exists for velocities and these already have the energy minimum at the desired regulation point ($v = 0$).

The situation is different in other domains.
For the first of the two simple RLC circuits considered previously, dissipation appears in a coordinate, x_2, which already has the minimum at the desired point.

For the second one, the minimum of the energy has to be moved for both coordinates, and hence the dissipation obstacle is unavoidable.