

Corrigendum to “On the control of non–linear processes: an IDA–PBC approach” (H. Ramírez et. al., *Journal of Proc. Cont.* 2009; 19:405-414)

Carles Batlle^a, Romeo Ortega^b,
Daniel Sbarbaro, Hector Ramírez^c

^a*Departament de Matemàtica Aplicada IV & IOC, Universitat Politècnica de Catalunya, EPSEVG, Av. V. Balaguer s/n, 08800 Vilanova i la Geltrú, Spain*

^b*Laboratoire des Signaux et Systèmes, Supelec, Plateau de Moulon, 91192 Gif Sur Yvette Cedex, France (Tel: +33 1 69851766)*

^c*Department of Electrical Engineering, Universidad de Concepción, Concepción, Chile (Tel: +56 41 2204353)*

Abstract

This note points out to some unfortunate errors contained in the paper [“On the control of non–linear processes: An IDA–PBC approach”, *Journal of Process Control*, Vol. 19, Issue 3, pp. 405-414, 2009], which invalidate the proposed technique to apply the Interconnection and Damping Assignment Passivity–based Control method in a non–matching setting.

In a recent paper [1], the Interconnection and Damping Assignment Passivity–based Control (IDA–PBC) method [2] is studied in the case when the key matching equation is not satisfied. The paper proposes to fix a desired storage function and to design a scaling matrix, which is obtained solving an algebraic Lyapunov equation, in such a way that the linearized dynamics of the

Email addresses: carles.batlle@upc.edu (Carles Batlle), ortega@lss.supelec.fr (Romeo Ortega), dsbarbar@udec.cl (Daniel Sbarbaro, Hector Ramírez).

closed-loop system is stable—without the need to solve the matching equation. Unfortunately, the derivations in [1] are incorrect, invalidating the results of the paper, in particular, Proposition 1.

To explain the problems encountered in [1], in the present note we first re-derive the closed-loop dynamics of a system controlled using the IDA-PBC method in the case when the matching condition is not satisfied, that is, equation (11) in [1]. This computations are carried-out following a route, which is slightly different from the one used in [1], that we believe is more general and straightforward. Then, we indicate the points that invalidate Proposition 1.

Consider an affine control system

$$\dot{x} = f(x) + g(x)u, \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m, \quad m < n, \quad (1)$$

and an assignable equilibrium that we want to stabilize $x^* \in \mathbb{R}^n$. Notice that, without loss of generality, we can assume that $\text{rank } g(x) = m$, which in turn implies that $\det(g^\top(x)g(x)) \neq 0$. Let $g^\perp(x) \in \mathbb{R}^{(n-m) \times n}$ denote a nontrivial, full-rank, left annihilator of $g(x)$, that is,

$$g^\perp(x)g(x) = 0, \quad \text{rank } g^\perp(x) = n - m. \quad (2)$$

Hence, one also has $\det(g^\perp(x)(g^\perp(x))^\top) \neq 0$.

In IDA-PBC one looks for a state feedback $u = \beta(x)$ such that the closed-loop dynamics takes the form [2]

$$\dot{x} = F_d(x)\partial H_d^\top(x) \quad (3)$$

with $F_d(x) \in \mathbb{R}^{n \times n}$, fixed a priori, and such that $F_d(x) + F_d^\top(x) \leq 0$, and with $H_d(x)$ a function to be defined such that it has a strict minimum at x^* .¹ Matching the right hand sides of (3) and (1), with $u = \beta(x)$, one gets

$$f(x) + g(x)\beta(x) = F_d(x)\partial H_d^\top(x). \quad (4)$$

Now, consider the $n \times n$ matrix

$$Q(x) = \begin{pmatrix} g^\perp(x) \\ g^\top(x) \end{pmatrix}. \quad (5)$$

This matrix is nonsingular because the rows of $g^\perp(x)$ are independent, the rows of $g^\top(x)$ are also independent and the rows of $g^\top(x)$ and $g^\perp(x)$ are mutually

¹ In [1], $F_d(x)$ is split into its skew-symmetric and symmetric, negative semidefinite parts as $F_d(x) = J_d(x) - R_d(x)$.

independent. Pre-multiplying (4) by (5) one gets the following two equations

$$\beta(x) = (g^\top(x)g(x))^{-1}g^\top(x) \left(F_d(x)\partial H_d^\top(x) - f(x) \right) \quad (6)$$

$$0 = g^\perp(x) \left(F_d(x)\partial H_d^\top(x) - f(x) \right) \quad (7)$$

which are, obviously, equivalent to (4). The first equation univocally defines the control signal, while the second one, called the *matching equation* (ME), is a partial differential equation in $H_d(x)$, whose solutions characterize the admissible energy functions that can be assigned to the closed-loop dynamics.

In [1] we were interested in the case when ME cannot be solved and studied the closed-loop dynamics that results from the application of (6) to (1), that is,

$$\dot{x} = f(x) + g^\top(x)(g^\top(x)g(x))^{-1}g^\top(x) \left(F_d(x)\partial H_d^\top(x) - f(x) \right). \quad (8)$$

Now, it is possible to show, see *e.g.*, [3], that

$$(g^\perp(x))^\top \left(g^\perp(x)(g^\perp(x))^\top \right)^{-1} g^\perp(x) + g(x) \left(g^\top(x)g(x) \right)^{-1} g^\top(x) = \mathbb{I}_n. \quad (9)$$

Using (9) in (8) yields the following form of the closed-loop dynamics

$$\dot{x} = F_d(x)\partial H_d^\top(x) - Q_1^{-1}(x)g^\perp(x) \left(F_d(x)\partial H_d^\top(x) - f(x) \right), \quad (10)$$

where, using the notation of [1], we have defined

$$Q_1^{-1}(x) := (g^\perp(x))^\top \left(g^\perp(x)(g^\perp(x))^\top \right)^{-1}, \quad (11)$$

which, as shown in [1] is made of the first $n - m$ columns of $Q^{-1}(x)$, hence the name. Equation (10) clearly displays the mismatch of the desired dynamics—along the directions perpendicular to $g(x)$ —when ME is not satisfied.

Clearly, (10) can be rewritten as

$$\dot{x} = \left(\mathbb{I}_n - Q_1^{-1}(x)g^\perp(x) \right) F_d(x)\partial H_d^\top(x) + Q_1^{-1}(x)g^\perp(x)f(x) =: f_{CLNM}(x), \quad (12)$$

where, for latter reference, we have defined the closed-loop non-matching equation vector field $f_{CLNM}(x)$. Now, in view of the conditions on $H_d(x)$ and the fact that x^* is an admissible equilibrium, one has

$$\partial H_d^\top(x^*) = 0, \quad g^\perp(x^*)f(x^*) = 0. \quad (13)$$

It follows from this that $f_{CLNM}(x)$ has $x = x^*$ as a fixed point and it makes sense to perform a local linear analysis of the closed-loop system around x^* . Defining $\tilde{x} := x - x^*$, and using (13), one has that the linear approximation of $f_{CLNM}(x)$ is given by

$$\dot{\tilde{x}} = (A + B)\tilde{x} + C\tilde{x}, \quad (14)$$

where

$$A := F_d(x^*)\partial^2 H_d(x^*), \quad B := -Q_1^{-1}(x^*)g^\perp(x^*)F_d(x^*)\partial^2 H_d(x^*), \quad (15)$$

$$C := Q_1^{-1}(x^*)\partial(g^\perp(x^*)f(x^*)).$$

Equation (14) is contained in Proposition 1 of [1], modulo a sign mistake in equation (17).

The main idea behind the derivations of [1] is to *construct* the function $H_d(x)$ as a scaled version of a separable function, see (13) and (14) in [1], in such a way as to be able to assign a quadratic Lyapunov function, (20) in [1], to (14). Unfortunately, two fatal errors slipped into the calculations. First, that the matrix $A + B$, is *not Hurwitz*, as required by the construction proposed in [1]. Indeed, this matrix, that may be written as

$$A + B = (\mathbb{I}_n - Q_1^{-1}(x^*)g^\perp(x^*))F_d(x^*)\partial^2 H_d(x^*) \quad (16)$$

has $g^\perp(x^*)$ as a left null vector, since

$$g^\perp(x)Q_1^{-1}(x)g^\perp(x) = g^\perp(x). \quad (17)$$

A second problem relates to the matrix manipulations leading to equation (15) in [1]. The gradient ∂H_d^T is written as

$$\partial H_d^T = P\partial\xi^T, \quad (18)$$

where ξ is taken as a sum of n functions, each one depending only on one of the states, and P is an unknown matrix for which the Lyapunov equation in [1] has to be solved. Unfortunately, invoking Poincare's lemma it is easy to see that this identity holds only if P is diagonal. Moreover, in order to obtain the equation in [1], P must be commuted with the Hessian matrix of ξ , and the conditions placed on the later do not warrant that operation, therefore leading to extra constraints on P which are, in general, incompatible with the Lyapunov equation that it should obey.

Thus, the P matrices in the examples are wrong. However, the simulations work since the proposed analysis, based on Lyapunov, provides only sufficient conditions, and therefore local stability can certainly be analyzed by other means such as checking the eigenvalues of $A + B + C$.

Before wrapping up the note we would like to bring to the readers' attention the recent results of [4]. The authors propose to solve the linearized matching equation, which turns out to be a linear matrix inequality (LMI). Moreover, they prove that, under some reasonable rank assumptions, the LMI is feasible if and only if the linearized system is stabilizable.

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