

## SIMULTANEOUS INTERCONNECTION AND DAMPING ASSIGNMENT PASSIVITY-BASED CONTROL: TWO PRACTICAL EXAMPLES<sup>1</sup>

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Abstract: We argue in this paper that the standard two-stage procedure used in Interconnection and Damping Assignment Passivity-Based Control (IDA-PBC)—consisting of splitting the control action into the sum of energy-shaping and damping injection terms—is not without loss of generality, and effectively reduces the set of systems that can be stabilized with IDA-PBC. To overcome this problem we suggest in this paper to carry out *simultaneously* both stages and refer to this variation of the method as SIDA-PBC. To illustrate the application of SIDA-PBC we consider two practically important examples: The torque and rotor flux regulation control of induction motors and the power flow regulation control of a doubly-fed induction generator. *Copyright ©2006 IFAC*

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### 1. INTRODUCTION

Passivity-based control (PBC) is a generic name given to a family of controller design techniques that achieves system stabilization via the route

of passivation, that is, rendering the closed-loop system passive with a desired storage function (that usually qualifies as a Lyapunov function for the stability analysis.) If the passivity property turns out to be output strict, with an output signal with respect to which the system is detectable, then asymptotic stability is ensured. See the monographs Ortega *et al.* (1998); van der Schaft (2000), and Ortega and Garcia-Canseco (2004) for a recent survey.

As is well-known, van der Schaft (2000), a passive system can be rendered strictly passive simply adding a negative feedback loop around the passive output—an action sometimes called  $L_gV$

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control, Sepulchre *et al.* (1997). For this reason, it has been found convenient in some applications, in particular for mechanical systems, Takegaki and Arimoto (1981), Ortega *et al.* (2002a), to split the control action into the sum of two terms, an energy-shaping term which, as indicated by its name, is responsible of assigning the desired energy/storage function to the passive map, and a second  $L_g V$  term that injects damping for asymptotic stability. The purpose of this paper is to bring to the readers attention the fact that splitting the control action in this way is not without loss of generality, and effectively reduces the set of problems that can be solved via PBC. This assertion is, of course, not surprising since it is clear that, to achieve strict passivity, the procedure described above is just one of many other possible ways. Our point is illustrated with the IDA-PBC design methodology proposed in Ortega *et al.* (2002). To enlarge the set of systems that can be stabilized via IDA-PBC we suggest to carry out *simultaneously* the energy shaping and the damping injection stages and refer to this variation of the method as SIDA-PBC.

We illustrate the application of SIDA-PBC with two practically important examples. First, we show that the fundamental problem of induction motor torque and rotor flux regulation *cannot be solved* with two stage IDA-PBC. It is, however, solvable with SIDA-PBC. Second, we prove that with SIDA-PBC we can shape the total energy of the *full (electrical and mechanical) dynamics* of a doubly-fed induction generator used in power flow regulation tasks while, as reported in Batlle *et al.* (2005), with two stage IDA-PBC only the electrical energy could be shaped. Simulation results of these examples are presented to illustrate the performance improvement obtained with SIDA-PBC.

## 2. PBC WITH SIMULTANEOUS ENERGY SHAPING AND DAMPING INJECTION

We consider the problem of stabilization of an equilibrium point for nonlinear systems of the form

$$\dot{x} = f(x, t) + g(x)u \quad (1)$$

where  $x \in \mathbb{R}^n$  is the state vector,  $u \in \mathbb{R}^m$ ,  $m < n$  is the control action and  $g(x)$  is assumed full rank. In *two-stage IDA-PBC* this objective is achieved as follows, see Ortega *et al.* (2002,a) for further details. First, decompose the control signal in two terms

$$u = u_{es} + u_{di} \quad (2)$$

where  $u_{es}$  is responsible of the energy-shaping stage and  $u_{di}$  injects the damping. Second, solve

the key *matching equation*<sup>2</sup>

$$g^\perp(x)f(x, t) = g^\perp(x)J_d(x, t)\nabla H_d \quad (3)$$

for some functions

$$J_d : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^{n \times n}, \quad H_d : \mathbb{R}^n \rightarrow \mathbb{R},$$

satisfying the skew-symmetry condition for the interconnection matrix

$$J_d(x, t) + J_d^\top(x, t) = 0, \quad (4)$$

and the equilibrium assignment condition for the desired total stored energy

$$x_\star = \arg \min H_d(x) \quad (5)$$

with  $x_\star \in \mathbb{R}^n$  the equilibrium to be stabilized<sup>3</sup> and  $g^\perp(x) \in \mathbb{R}^{(n-m) \times n}$  a full-rank left-annihilator of  $g(x)$ , that is,  $g^\perp(x)g(x) = 0$  and  $\text{rank } g^\perp(x) = n - m$ .

As shown in Ortega *et al.* (2002) system (1) in closed-loop with the control (2), with

$$u_{es} = [g^\top(x)g(x)]^{-1}g^\top(x)\{J_d(x, t)\nabla H_d - f(x, t)\}. \quad (6)$$

yields a port-controlled Hamiltonian (PCH) system of the form

$$\begin{aligned} \dot{x} &= J_d(x, t)\nabla H_d + g(x)u_{di} \\ y &= g^\top(x)\nabla H_d. \end{aligned} \quad (7)$$

The system (7) without damping injection term is conservative, i.e.,  $\dot{H}_d = 0$ , with  $x_\star$  a stable equilibrium (with Lyapunov function  $H_d(x)$ ). To *add dissipation* we feedback the passive output  $y$ , for instance, with

$$u_{di} = -K_{di}y, \quad K_{di} = K_{di}^\top > 0,$$

to finally obtain the PCH system with dissipation

$$\begin{aligned} \dot{x} &= [J_d(x, t) - R_d(x)]\nabla H_d + g(x)v \\ y &= g^\top(x)\nabla H_d. \end{aligned} \quad (8)$$

where the damping matrix  $R_d(x) = R_d^\top(x) \geq 0$  is defined by

$$R_d(x) = g(x)K_{di}g^\top(x),$$

and we have added a signal  $v$  to (2) to define the port variables. Since the new closed-loop system (with  $v = 0$ ) satisfies  $\dot{H}_d = -y^\top K_{di}y$ , it is easy to prove (e.g., Lemma 3.2.8 of van der Schaft (2000)) that the equilibrium  $x_\star$  will now be asymptotically stable if it is detectable from  $y$ , i.e., if the implication  $(y(t) \equiv 0 \Rightarrow \lim_{t \rightarrow \infty} x(t) = x_\star)$  is true.

Obviously, the key for the success of IDA-PBC is the solution of the matching equation (3). With

<sup>2</sup> All vectors in the paper are *column* vectors, even the gradient of a scalar function denoted  $\nabla_{(\cdot)} = \frac{\partial}{\partial(\cdot)}$ . When clear from the context the subindex will be omitted.

<sup>3</sup> That is,  $x_\star$  is a member of the set  $\{\bar{x} \in \mathbb{R}^n \mid g^\perp(\bar{x})f(\bar{x}, t) = 0, \forall t \in \mathbb{R}\}$ .

the motivation of enlarging the class of systems for which this equation is solvable we propose in this paper to avoid the decomposition of the control into energy–shaping and damping injection terms. Instead, we suggest to carry out *simultaneously* both stages and replace (3), with the SIDA–PBC matching equations

$$g^\perp(x)f(x,t) = g^\perp(x)F_d(x,t)\nabla H_d, \quad (9)$$

to replace the constraint (4) by the *strictly weaker* condition

$$F_d(x,t) + F_d^\top(x,t) \leq 0, \quad (10)$$

and define the control as

$$u = [g^\top(x)g(x)]^{-1}g^\top(x)\{F_d(x,t)\nabla H_d - f(x,t)\}.$$

Since the set of skew–symmetric matrices is strictly contained in the set of matrices with negative semi–definite symmetric part, it is clear that the set of functions  $\{f(x,t), g(x)\}$  for which (3)—subject to the constraint (4)—is solvable is strictly smaller than the set for which (9), subject to (10), is solvable.

*Remark 1.* There exists several techniques to solve the matching equations (3) (resp., (9)), with two extremes being the purely algebraic approach of Fujimoto and Sugie (2001) and the PDE approach of Ortega *et al.* (2002). In the former  $H_d(x)$  is *a priori fixed*, which makes (3) (resp., (9)) an algebraic equation that is solved for  $J_d(x,t)$  (resp.,  $F_d(x,t)$ )—subject to the constraint (4) (resp., (10)). On the other hand, in the latter  $J_d(x,t)$  (resp.,  $F_d(x,t)$ ) is fixed making (3) (resp., (9)) a PDE that is solved for  $H_d(x)$ . We refer the interested reader to Ortega and Garcia-Canseco (2004) for a detailed discussion on these, as well as other, methods of solution of the matching equations. In this paper we will adopt the algebraic approach.

*Remark 2.* Similarly to IDA–PBC, application of SIDA–PBC also yields a closed–loop PCH system of the form (8) with

$$J_d(x,t) = \frac{1}{2}[F_d(x,t) - F_d^\top(x,t)],$$

$$R_d(x,t) = \frac{1}{2}[F_d(x,t) + F_d^\top(x,t)].$$

*Remark 3.* To make IDA–PBC applicable to non–autonomous systems, which will be required in the induction motor application, we have presented above a slight variation of the method. Notice that the matrices  $J_d$  and  $R_d$  may depend explicitly on time. Clearly, their skew–symmetry and non–negativity properties must now hold uniformly in time as well.

### 3. INDUCTION MOTOR CONTROL VIA SIDA–PBC

In this section we will show that the problem of output feedback torque control of induction motors is *not solvable* via two–stage IDA–PBC but it is solvable with SIDA–PBC. An interesting feature of our SIDA–PBC is that we establish here (Lyapunov) stability of a *given equilibrium* that generates the desired torque and rotor flux amplitude.

The standard two–phase model, which rotates at an arbitrary speed  $\omega_s \in \mathbb{R}$ , is given by Krause *et al.* (1995)

$$\begin{aligned} \dot{x}_{12} = & -[\gamma I_2 + (n_p\omega + u_3)\mathcal{J}]x_{12} \\ & + \alpha_1(I_2 - T_r n_p \omega \mathcal{J})x_{34} + \alpha_2 u_{12} \end{aligned} \quad (11)$$

$$\dot{x}_{34} = -\left(\frac{1}{T_r}I_2 + \mathcal{J}u_3\right)x_{34} + \frac{L_{sr}}{T_r}x_{12} \quad (12)$$

$$\dot{\omega} = \alpha_3 x_{12}^\top \mathcal{J} x_{34} - \frac{\tau_L}{J_m} \quad (13)$$

in which  $I_2 \in \mathbb{R}^{2 \times 2}$  is the identity matrix,  $\mathcal{J} = -\mathcal{J}^\top \in \mathbb{R}^{2 \times 2}$ ,  $x_{12} \in \mathbb{R}^2$  are the stator currents,  $x_{34} \in \mathbb{R}^2$  the rotor fluxes,  $\omega \in \mathbb{R}$  the rotor speed,  $u_{12} \in \mathbb{R}^2$  are the stator voltages,  $\tau_L \in \mathbb{R}$  is the load torque and  $u_3 := \omega_s - n_p\omega$ . All the parameters are positive and defined in the usual way. Notice that, as first pointed out in the control literature in Ortega and Espinosa (1993), the signal  $u_3$  acts as an additional control input. We will select  $u_3$  to transform the periodic orbits of the system into constant equilibria.

We are interested in this paper in the problem of regulation of the motor torque and the rotor flux amplitude

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} J_m \alpha_3 x_{12}^\top \mathcal{J} x_{34} \\ |x_{34}| \end{bmatrix}, \quad (14)$$

to some constant desired values  $y_\star = \text{col}(y_{1\star}, y_{2\star})$ , where  $|\cdot|$  is the Euclidean norm, assuming that the only signals *available for measurement* are  $x_{12}$  and  $\omega$ .

To solve this problem using (S)IDA–PBC it is necessary to express the control objective in terms of a *desired equilibrium*. In this sense, it can be shown that for the induction motor model (11)–(13) with output functions (14) and

$$u_3 = u_{3\star} := \frac{R_r}{n_p} \frac{y_{1\star}}{y_{2\star}^2}. \quad (15)$$

the set of assignable equilibrium points, denoted  $\text{col}(\bar{x}_{12}, \bar{x}_{34}, \bar{\omega}) \in \mathbb{R}^5$ , which are compatible with the desired outputs  $y_\star$  is defined by

$$\bar{x}_{12} = \frac{1}{L_{sr}} \begin{bmatrix} 1 & -\frac{L_r y_{1\star}}{n_p y_{2\star}^2} \\ \frac{L_r y_{1\star}}{n_p y_{2\star}^2} & 1 \end{bmatrix} \bar{x}_{34}$$

$$|\bar{x}_{34}| = y_{2\star} \quad (16)$$

with  $\bar{\omega}$  arbitrary.

*Remark 4.* From (13) and (14) we see that to operate the system in equilibrium,  $y_{1\star} = \tau_L$ —hence, to define the desired equilibrium the load torque *needs to be known*. In practical applications, an outer loop PI control around the velocity error is usually added. The output of the integrator, on one hand, provides an estimate of  $\tau_L$  while, on the other hand, ensures that speed also converges to the desired value. A scheme that removes this assumption has recently been proposed in Karagiannis *et al.* (2005).

As indicated in Remark 1, in this paper we will adopt the *algebraic approach* to solve the matching equations. To this end, we will consider for simplicity a quadratic in errors energy function of the form

$$H_d(x) = \frac{1}{2}(x - x_\star)^\top P(x - x_\star), \quad (17)$$

with  $P = P^\top > 0$  a matrix to be determined. Moreover, the problem formulation is simplified by using the generic symbol  $F(x, t)$  to denote either  $J_d(x, t)$  or  $F_d(x, t)$ , and identifying the IDA and SIDA approaches by imposing either  $F(x, t) + F^\top(x, t) = 0$  or  $F(x, t) + F^\top(x, t) \leq 0$ , respectively.

Since we are interested here in torque control, and this is only defined by the stator currents and the rotor fluxes, its regulation can be achieved applying IDA–PBC *to the electrical subsystem only*. Boundedness of  $\omega$  will be established in a subsequent analysis.

If the electrical subsystem (11), (12), with  $u_3 = u_{3\star}$  and  $u = u_{12}$ , is written in the form (1), then, considering that  $g^\perp = [0_{2 \times 2} \ I_2]$ , it is possible to notice that the matching equations (3) and (9) concern only the third and fourth rows of  $f(x, t)$  and they take the form

$$-\frac{1}{T_r} I_2 + \mathcal{J} u_{3\star} x_{34} + \frac{L_{sr}}{T_r} x_{12} = [F_3(x, t) \ F_4(x, t)] P(x - x_\star), \quad (18)$$

where, to simplify the notation, we partition  $F(x, t)$  into  $F_i \in \mathbb{R}^{2 \times 2}$ ,  $i = 1, \dots, 4$ , sub-matrices.

On the other hand, considering the constraint imposed by the possibility of measuring only  $x_{12}$  and  $\omega$ , from (6) it is possible to see that the control can be written, factoring the components that depend on the unmeasurable quantity  $x_{34}$ , as

$$u_{12} = \hat{u}_{12}(x_{12}, \omega) + \frac{1}{\alpha_2} S(x, t) x_{34}$$

with  $\hat{u}_{12}(x_{12}, \omega)$  given in (21). Hence, it is clear that to verify the output feedback condition it must be satisfied that

$$S(x, t) := \alpha_1 [T_r n_p \omega(t) \mathcal{J} - I_2] + F_1(x, t) P_2 + F_2(x, t) P_3 = 0, \quad (19)$$

where we have partitioned the symmetric matrix  $P$  into  $P_i \in \mathbb{R}^{2 \times 2}$ ,  $i = 1, 2, 3$ , sub-matrices.

In order to show that the energy–shaping problem is *not solvable*, it is possible to establish that condition  $F(x, t) + F^\top(x, t) = 0$  is equivalent to

$$G = P_3^{-1} (-G^\top) P_3.$$

where

$$G := \frac{1}{\lambda} \left[ \frac{L_{sr}}{T_r} P_3 + P_1 (\alpha_1 I_2 - \lambda \beta_2 \mathcal{J}) \right]$$

with  $\lambda \in \mathbb{R}$ ,  $\lambda \neq 0$ ,  $\beta_2 \in \mathbb{R}$  and  $P_3$  a full rank matrix.

Consequently,  $G$  must be similar to  $-G^\top$ , and both necessarily have the same eigenvalues. A necessary condition for the latter is that  $\text{trace}(G) = 0$ , that clearly is not satisfied.

On the other hand, it can be shown that  $P_2 = 0$ ,  $F_2(t) = \alpha_1 [I_2 - T_r n_p \omega(t) \mathcal{J}] P_3^{-1}$ ,  $F_3 = \frac{L_{sr}}{T_r} P_1^{-1}$ ,  $F_4 = -\left(\frac{1}{T_r} I_2 + u_{3\star} \mathcal{J}\right) P_3^{-1}$  with  $F_1(x, t), P_1, P_3$  free, provide a solution to (18) and (19). Moreover, if  $P_1 = \frac{L_{sr}}{T_r} I_2$ ,  $P_3 = \alpha_1 I_2$  and  $F_1(t) = -\mathcal{K}(\omega(t))$ , with

$$\mathcal{K}(\omega(t)) > \frac{L_{sr}}{L_s L_r - L_{sr}^2} \left[ 1 + \frac{1}{4} (T_r n_p \omega(t))^2 \right] I_2. \quad (20)$$

then  $F(t) + F^\top(t) < 0$ .

The final part of the design is the explicit definition of the resulting controller, given by  $u_3 = \frac{R_r y_{1\star}}{n_p y_{2\star}^2}$  and  $u_{12} = \hat{u}_{12}(x_{12}, \omega)$  with

$$\begin{aligned} \hat{u}_{12}(x_{12}, \omega) = & \frac{1}{\alpha_2} [\gamma I_2 + (n_p \omega + u_{3\star}) \mathcal{J}] x_{12} \\ & - \frac{\alpha_1}{\alpha_2} (I_2 - T_r n_p \omega \mathcal{J}) x_{34\star} \\ & - \frac{L_{sr}}{\alpha_2 T_r} \mathcal{K}(\omega) (x_{12} - x_{12\star}) \end{aligned} \quad (21)$$

and  $\mathcal{K}(\omega)$  satisfying (20), which guarantees that the equilibrium  $x_\star$  is *globally exponentially stable* while  $\omega$  remains bounded.

The performance of the proposed SIDA–PBC was investigated by simulations. The considered motor parameters were taken from Ortega and Espinosa (1993). The rotor flux equilibrium value was set to  $x_{34\star} = \text{col}(\beta, 0)$  with  $\beta = 2$ , while  $x_{12\star}$  where computed according to (16). In the experiment, with the motor at standstill and a startup zero load torque, a profile for this latter variable was considered going first to  $\tau_L = 20Nm$  and later on

to  $\tau_L = 40Nm$ . In Figure 1 it is shown how the generated torque regulation objective is achieved.

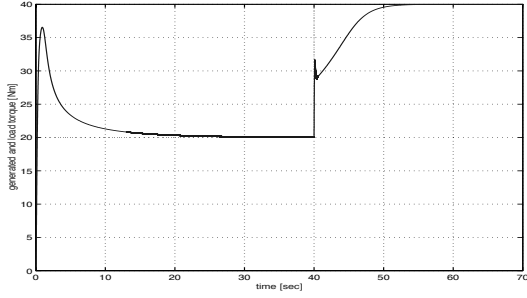


Fig. 1. Generated and load torque of the IM

#### 4. TOTAL ENERGY-SHAPING OF A DOUBLY-FED INDUCTION GENERATOR

The second considered example in this paper is related with the control of a doubly-fed induction machine (DFIM) Peresada *et al.* (2004). In this case the device acts as an energy-switching device between a local energy storing element (a flywheel) and the electrical power network. The control objective is to change the direction of the power flow (towards or from the flywheel) depending on the load demand. In Batlle *et al.* (2005) the equilibria associated to these regimes is stabilized with IDA-PBCs that shape the *electrical energy*, treating the mechanical dynamics as a cascaded subsystem. The purpose of this section is to show that using SIDA-PBC it is possible to shape the energy function of the *complete* system dynamics, resulting in a controller with improved power-flow regulation performance due to the possibility of considering a fast response of the mechanical speed. To the best of our knowledge, this is the first control algorithm for this class of systems that provides for this additional degree of freedom.

We consider the configuration for the DFIM studied in Batlle *et al.* (2005) where a representation in the  $dq$  framework rotating at the (constant) angular speed of the AC source ( $\omega_s$ ) is assumed. The energy function of the overall system is  $H(z) = \frac{1}{2}z^\top \mathcal{L}^{-1}z$ , and the model given by

$$\dot{z} = [J(i_s, \omega) - R]\nabla H + \begin{bmatrix} v_s \\ O_{2 \times 1} \\ \tau_L \end{bmatrix} + \begin{bmatrix} O_{2 \times 2} \\ I_2 \\ O_{2 \times 2} \end{bmatrix} u \quad (22)$$

where  $v_s \in \mathbb{R}^2$ ,  $\tau_L \in \mathbb{R}$  are, respectively, the stator voltage and the external mechanical torque, which are *constant*,  $u \in \mathbb{R}^2$  are the rotor *control* voltages,  $\mathcal{L} = \mathcal{L}^\top > 0$  is the generalized inductance matrix and  $z = \text{col}(\lambda_s, \lambda_r, J_m \omega) \in \mathbb{R}^5$ , where  $\lambda_s, \lambda_r \in \mathbb{R}^2$  are the stator and rotor fluxes, respectively, and  $\omega \in \mathbb{R}$  is the mechanical speed. The (skew-symmetric) structure and damping matrices are

$$J(i_s, \omega) := \begin{bmatrix} -\omega_s L_s \mathcal{J} & -\omega_s L_{sr} \mathcal{J} & O_{2 \times 1} \\ -\omega_s L_{sr} \mathcal{J} & -(\omega_s - \omega) L_r \mathcal{J} & L_{sr} \mathcal{J} i_s \\ O_{1 \times 2} & L_{sr} i_s^\top \mathcal{J} & 0 \end{bmatrix},$$

$$R := \begin{bmatrix} R_s I_2 & O_{2 \times 2} & O_{2 \times 1} \\ O_{2 \times 2} & R_r I_2 & O_{2 \times 1} \\ O_{1 \times 2} & O_{1 \times 2} & B_r \end{bmatrix} > 0,$$

All machine parameters are defined in the usual way. Also note that the vector  $\chi = \text{col}(i_s, i_r, \omega) \in \mathbb{R}^{5 \times 5}$ , where  $i_s, i_r \in \mathbb{R}^2$  are the stator and rotor currents, respectively, satisfies that  $z = \mathcal{L}\chi$ .

Clearly, the fixed point equations for (22) are given by  $z_\star = \mathcal{L}\chi_\star$ , with  $\chi_\star := \text{col}(i_{s\star}, i_{r\star}, \omega_\star)$  the solutions of

$$\begin{aligned} -(\omega_s L_s \mathcal{J} + R_s I_2) i_{s\star} - \omega_s L_{sr} \mathcal{J} i_{r\star} + v_s &= 0 \\ L_{sr} i_{s\star}^\top \mathcal{J} i_{r\star} - B_r \omega_\star + \tau_L &= 0, \end{aligned} \quad (23)$$

As discussed in Batlle *et al.* (2005), the direction of the power flow can be regulated commuting between two controllers that stabilize two different equilibrium points.

In order to obtain the controller that stabilizes the desired equilibrium, as done in Section 3 we will design our SIDA-PBC adopting the algebraic approach. For, we fix the desired energy function as

$$H_d(z) = \frac{1}{2}(z - z_\star)^\top P(z - z_\star), \quad P = P^\top > 0. \quad (24)$$

Thus SIDA-PBC design reduces to finding a matrix  $F_d(z)$  such that the right-hand term of (22) equals  $F_d(z)P(z - z_\star)$  and verifying  $F_d(z) + F_d^\top(z) \leq 0$ . To simplify the solution we restrict  $P$  to be diagonal while

$$F_d(z) = \begin{bmatrix} F_{11}(z) & F_{12}(z) & O_{2 \times 1} \\ F_{21}(z) & F_{22}(z) & F_{23}(z) \\ F_{31}^\top(z) & F_{32}^\top(z) & F_{33}(z) \end{bmatrix},$$

It can be shown that if  $F_{11} = -\frac{1}{p_s} \left( \omega_s \mathcal{J} + \frac{L_r}{\mu} R_s I_2 \right)$  and  $F_{12} = \frac{L_{sr}}{p_r \mu} R_s I_2$  with  $\mu := L_s L_r - L_{sr}^2 > 0$ , with  $F_{31} = \frac{L_{sr}}{p_s \mu} \mathcal{J} \lambda_{r\star}$ ,  $F_{32}(z) = -\frac{L_{sr}}{p_r \mu} \mathcal{J} \lambda_s$ ,  $F_{33} = -\frac{B_r}{p_\omega J_m}$ , while

$$F_{21} = -F_{12}, \quad F_{23}(z) = -F_{32}(z), \quad F_{22} = -\frac{k_r}{2p_r} I_2 < 0$$

yields

$$F_d(z) + F_d^\top(z) = \begin{bmatrix} -\frac{2L_r R_s}{p_s \mu} I_2 & O_{2 \times 2} & \frac{L_{sr}}{p_s \mu} \mathcal{J} \lambda_{r\star} \\ O_{2 \times 2} & -\frac{k_r}{p_r} I_2 & O_{2 \times 1} \\ -\frac{L_{sr}}{p_s \mu} \lambda_{r\star}^\top \mathcal{J} & O_{1 \times 2} & -\frac{2B_r}{p_\omega J_m} \end{bmatrix}. \quad (25)$$

A simple Schur's complement analysis establishes that  $F_d(z) + F_d^\top(z) < 0$  if and only if the *free parameters*  $p_s$  and  $p_\omega$  satisfy

$$p_s > \left( \frac{J_m L_{sr}^2}{4B_r L_r R_s \mu} |\lambda_{r\star}|^2 \right) p_\omega. \quad (26)$$

Once we have solved the SIDA–PBC matching equations, the design is completed computing the controller which in this case is given as the static feedback control

$$u = R_r \dot{i}_r + (\omega_s - \omega) \mathcal{J}(L_{sr} \dot{i}_s + L_r \dot{i}_r) - k_s (L_s \tilde{i}_s + L_{sr} \tilde{i}_r) - k_r (L_{sr} \tilde{i}_s + L_r \tilde{i}_r) + k_\omega \mathcal{J} \lambda_s \tilde{\omega} \quad (27)$$

where  $k_r > 0$ ,  $k_\omega > 0$  and  $k_s > \frac{L_{sr}^2}{4B_r L_r \mu} |\lambda_{r*}|^2 k_\omega$ . Considering this control law, the equilibrium  $z_*$  is globally exponentially stable.

The usefulness of the controller (27) was illustrated by some simulations using the DFIM parameters of Batlle *et al.* (2005). The controller parameters were selected as  $k_s = 1000$ ,  $k_r = 100$  and  $k_\omega = 0.01$ . In Figure 2 we compare the speed behavior of the new SIDA–PBC with the IDA–PBC reported in Batlle *et al.* (2005)—that shapes only the electrical energy. As can be noticed, SIDA–PBC achieves a much faster speed response.

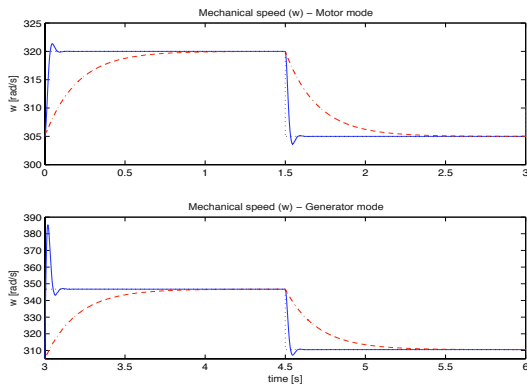


Fig. 2. Mechanical speed,  $\omega$ , for SIDA–PBC (continuous line) and IDA–PBC (dashed line).

## 5. CONCLUSIONS

We have presented an extension of the highly successful IDA–PBC methodology, called SIDA–PBC, where the energy–shaping and damping injection tasks are not performed sequentially, but simultaneously. In this way we enlarge the class of systems that can be stabilized using PBC and, furthermore, through the consideration of a broader set of desired damping matrices, we provide the designer with more tuning knobs to improve performance.

This new idea has been applied to solve the long standing problem of IDA–PBC of induction motors, that turns out to be unsolvable with a two stage design. Also, by avoiding the classical nested–loop control configuration prevalent in electromechanical systems, we have been able to improve the mechanical response of a DFIM,

working both as a motor and a generator. Experimental validation of the two control algorithms is currently being terminated and will be reported in the near future.

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